MULTI-SCALE MODELING OF FATIFUE DAMAGE OF CONTINUOUS FIBER REINFORCED POLYMER COMPOSITES

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Abstract Knowledge of the fatigue damage has been of great interest of designers of the composites. Here phenomenological and micromechanical approaches were developed. Former is based on the continuous damage mechanics (CDM) model assumes that plies are continuous. Damage tensor describes by three damage variables associated with matrix/fiber debonding, fiber breakage and transverse plies cracking. Stress-based constitutive model uses damage surface which is defined in terms of thermodynamic forces tensor driven by damage. The latter approach represents composite from microscopic point of view, i.e. consider two constituents unit cell (matrix and fiber). Polymer matrix has a non-linear behavior due to isotropic damage. Initiation criterion built on maximum principal stress and maximum shear stress criteria. Both constitutive models were developed in FORTRAN code for ABAQUS user subroutine UMAT. To validate models number of numerical simulation were performed. Phenomenological concept has been applied to 3D biaxial geometry. During microscopic analysis the unit and cross-ply unit cells were simulated.

1 INTRODUCTION

Nowadays composites materials have many applications in industry and particularly replaced metals, due to the good compromise between weight and resistance. Composites subjected to repetitive loading during exploitation that could lead to damage and as results to failure of structure. In recent decades a lot of models have been developed in order to describe material behavior. Nevertheless, still the fatigue of composites has been of researcher's interest [1].

Different approaches are presented in the literature and could be classified on three groups as: empirical, phenomenological and micromechanics. The new formulation of continuum damage mechanics was discussed in [2] where authors considered the basic ideas what should be involved to developed model. This model was conceptualized and applied for ceramic composites.

In [3] a new model was presented for fatigue damage evolution of polymer matrix composites. Orthotropic damage model was used for evaluation damage within cycle. For damage evolution author used thermodynamic forces potential which in case when damage depends on material's orientation could be represented by Tsai-Wu criterion.

Supporting the concept of non-linearity of a fiber-reinforced laminated composite due to damage in [4] authors considered main composites failure modes as: fiber breakage, matrix cracking, fiber/matrix debonding, and delamination/sliding. Critical values of thermodynamic forces were evaluated based on the tests data available in the literature.

In [5] authors considered total strain as composition of plastic, damage and elastic. This method based on strong coupling between damage and plasticity. Plastic dissipation potential expressed in terms of effective stresses. Damage dissipation potential has general form as plastic one. Authors revealed that in case of multiaxial loading the residual strain are decomposed on two term plastic and damage.

Here modified life prediction approach (SA) was developed based on continuous damage mechanics and micromechanics of composites and simplified approach of life prediction of the structure which are subjected to high cycles fatigue loading. Constitutive model is based on strong coupling plasticity-damage formulation and consider different types of interactions. Flow rules for damage and plasticity depend on both damage and plasticity potentials. Plasticity yield surface is described by von Mises-type with kinematic hardening and depends on the hydrostatic pressure. The damage yield surface is smooth quadratic surface defined in space of thermodynamic forces driven by damage and takes into account different types of damage interactions.

Whereupon we can deal with all possible non-linear behaviors due to pure plasticity/damage or damage strongly coupled with plasticity. This model can be applied for different material such as metal alloys or composites materials.

2 CONTINUOUS NON-INEAR APPROACH

In this section we shall consider theoretical basis of the plasticity coupled with continuous damage mechanics of composites. But at the beginning accepted assumption should be described. As we working on high cycles fatigue we accept that at macro level material behaves as linear but at micro is non-linear as a result of plasticity and damage. To model plasticity and damage mechanisms two potentials were predefined. In this case the evolution of the plastic deformation defined by plasticity potential and evolution of damage by thermodynamic forces associated with. Residual strains separated in two part one due to plasticity another due to damage.

$$\varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^d \qquad (1)$$

2.1 Plasticity

The plastic yield surface is von Mises form and expressed in effective stress formulation space at the micro level and written as:

$$f = \sqrt{(\tilde{\mathbf{s}} - \tilde{\mathbf{x}})(\tilde{\mathbf{s}} - \tilde{\mathbf{x}})} - \sigma_y - \tilde{k}(p) \qquad (2)$$

where $\tilde{\mathbf{s}} = \tilde{\boldsymbol{\sigma}} - \frac{1}{3} tr\tilde{\boldsymbol{\sigma}}\mathbf{I}$ – deviator of effective stress tensor; σ_y – elastic limit; $\tilde{\mathbf{x}}$ – kinematic hardening function; $\tilde{k}(\mathbf{p})$ – isotropic hardening; $p = \sqrt{\frac{3}{2}\varepsilon^p \varepsilon^p}$ – equivalent plastic strain.
Evolution equation is defined as:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}^p \frac{\partial f}{\partial \boldsymbol{\sigma}} \qquad (3)$$

Macroscopic stress tensor decomposed into microscopic component and residual stresses:

$$\boldsymbol{\Sigma} = \boldsymbol{\sigma} + \boldsymbol{\rho} \qquad (4)$$

Residual stress tensor is self-equilibrated and following condition should be satisfied:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\rho} = 0 \qquad (5)$$

Inelastic strains in composites as results of the plastic deformation of the matrix and correlating to residual stress tensor as following:

$$\boldsymbol{\rho} = \mu \boldsymbol{\varepsilon}^p \qquad (6)$$

2.2 Damage evolution

Damage processes in composites are initiation and growth of defects as fiber breakage, matrix cracking, fiber/matrix debonding, and delamination/sliding. In continuous damage mechanics damage introduced as macroscopic measurement of the microscopic defect. Effective stress tensor can be written as:

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{M}(\boldsymbol{d}) : \boldsymbol{\sigma} \qquad (7)$$

where $\boldsymbol{M}(\boldsymbol{d})$ – damage effect tensor of order four.
Thermodynamic forces driven by damage are defined as second-order tensor [6]:

$$\boldsymbol{y} = \boldsymbol{\sigma} : \left[\boldsymbol{C}^{-1} : \boldsymbol{M}(\boldsymbol{d})^{-1} : \frac{\partial \boldsymbol{M}(\boldsymbol{D})}{\partial \boldsymbol{D}}\right] : \boldsymbol{\sigma} \qquad (8)$$

Different forms of and ways of construction symmetric $\boldsymbol{M}(\boldsymbol{d})$ were discussed in [6,7]. Further in each particular case we shall define the damage effect tensor and effective stiffness tensor. Damage potential is described by quadratic surface as follows:

$$g = \sqrt{(\boldsymbol{y} - \boldsymbol{h})(\boldsymbol{y} - \boldsymbol{h})} - \gamma_0 - k_d(r) \qquad (9)$$

where $\boldsymbol{h}$ – kinematic hardening; γ_0 – damage threshold; $k_d(r)$ – isotropic hardening depends on equivalent strain $r = \sqrt{\varepsilon^d \varepsilon^d}$.

Evolution equation is defined as:

$$\dot{\boldsymbol{d}} = \dot{\lambda}^d \frac{\partial g}{\partial \boldsymbol{y}} \qquad (10)$$

2.3 SA for life prediction

In high cycles fatigue (HCF) life of the composites is within 10^6-10^7 cycles. In order to make SA applicable to HCF composites it has to be assumed that that fatigue limits could be defined. That means we have accepted possible damage could be observed but quantitatively not exceed predetermined value. This condition was dictated by fact that it was shown that during giga-cycle regimes degradation is continue and fatigue limits will not be the same [8].

SA was characterized as particular direct method [9] and has obvious advantages in comparison with step-by-step method when the shakedown analysis is performed. According to statement of the problem the numeric program was developed. The program is post-treatment software dealing with data given after performed analysis in ABAQUS. Input values are stress tensor components in each node at every time increments. The results of program's calculation are plastic and residual damage deformations and number cycles to failure. SA based on numerical procedure [10].

The calculation procedure could be schematically interpreted as shown on the Figure 1. Following main idea of this method is implementation of new variable named as y_p-space has been used instead of stress deviator space and y_d-space instead of thermodynamic forces for plastic and damage potentials accordingly. These variables represent residuals stresses. e_p and e_d are normal to plastic and damage potential surfaces accordingly. Upper indexes show step by time during one cycle.

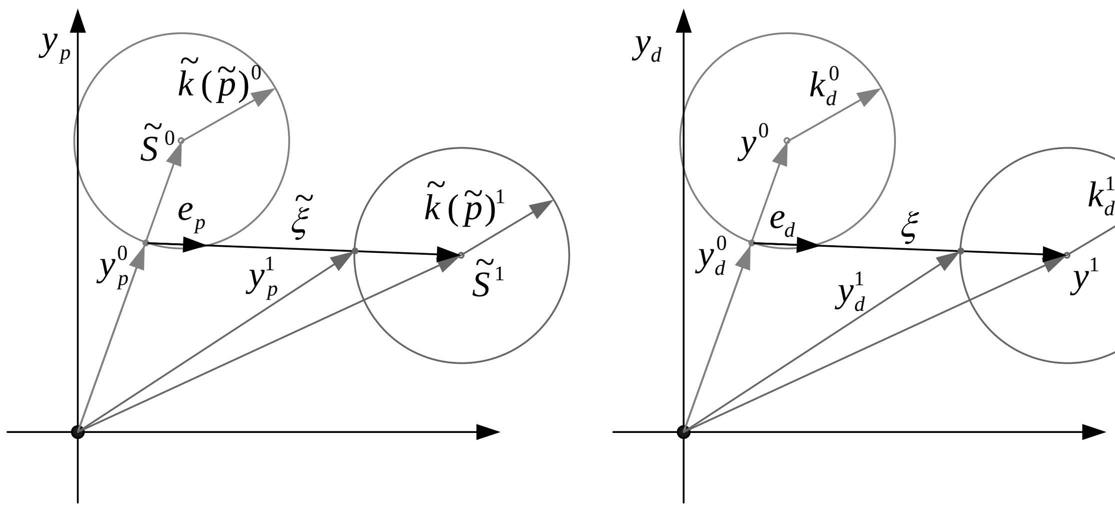

Figure 1. – Scheme of the simplified analysis scheme.

For developed procedure is enough performing one elastic finite element analysis and using this stress response we obtain life and inelastic strains. As a result significant reduction of computational costs could be achieved.

3 MULTI-SCALE APPROACH

Heretofore we supposed that stress localization tensor is equal to four order unit tensor and stress equation was written in simplify form as was shown in (4). Here we shall describe the application of the numerical multi-scale analysis which can be performed in order to take into account microscopic structure.

As well known multi-scale computation allows us to simulate heterogeneity of structures and define constitutive material model only at the micro-scale [11]. At macro-level geometry and boundary condition have to be specified. During calculation at each integration point at each time increment the micro-level geometry is called to evaluate tangent matrix and stress increment.

The price for that is a significant computational cost of calculation. Usage of the substructure modeling techniques highly reduces of the computational costs because substructure has been calculated just ones, and retained nodes-levels is representing unit cell model. This scheme is flexible and in easy way different unit-cell models can be used. In this works the simple unit-cells have been considered.

On Figure 2 general flow chart of transferring data from macro level to micro and back is shown. Where F, D , V and σ are respectively the deformation gradient tensor, tangent tensor, volume of unit cell and Cauchy stress tensor.

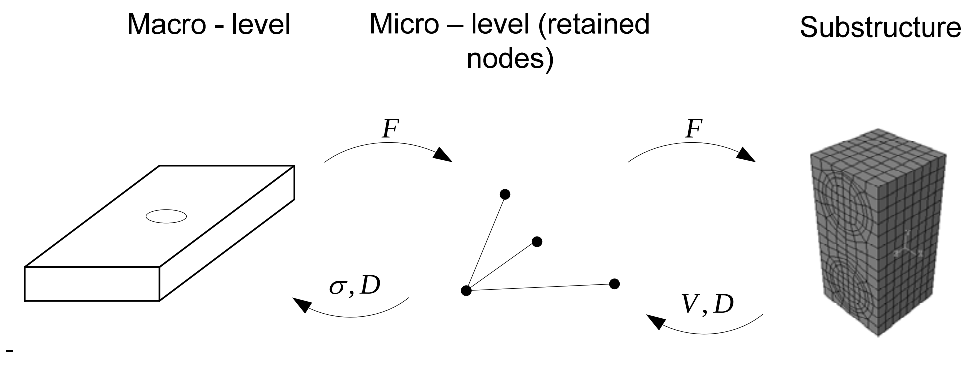

Figure 2. – Multi-scale scheme.

4 IMPLEMENTATION AND SIMULATION

4.1 UMAT subroutine

The constitutive model was implemented in UMAT subroutine Code of user subroutine can be used with three-dimensional solid elements or plane. Main steps of the procedure as following:

1. Strain increment was applied in the beginning;
2. Arraying stiffness matrix;
3. Checking of damage initiation criteria;
4. Propagation criteria;
5. Arraying effective stiffness matrix;
6. Update stresses;
7. Finding of a Jacobian matrix;
8. Finding of elastic energy.

4.2 Microscopic level

Damage simulation of simple unit cell model is shown on the Figure 3. Effective stiffness tensor depends on two damage variable: d_n which is characteristic of normal stiffness components and d_s - shear components. Stiffness tensor in Voigt notation is written as:

$$C = S^{-1} = \begin{bmatrix} \frac{1}{E(1-d_n)} & \frac{-\upsilon}{E(1-d_n)} & \frac{-\upsilon}{E(1-d_n)} & & & \\ \frac{-\upsilon}{E(1-d_n)} & \frac{1}{E(1-d_n)} & \frac{-\upsilon}{E(1-d_n)} & & & \\ \frac{-\upsilon}{E(1-d_n)} & \frac{-\upsilon}{E(1-d_n)} & \frac{1}{E(1-d_n)} & & & \\ & & & \frac{1}{G(1-d_s)} & & \\ & & & & \frac{1}{G(1-d_s)} & \\ & & & & & \frac{1}{G(1-d_s)} \end{bmatrix} \tag{11}$$

Table 1. Properties of micro-constituents [12].

Name	Matrix	Fiber
Longitudinal modulus, E1 (GPa)	80	3.35
Poisson’s ratio	0.2	0.35
Shear modulus (GPa)	33.33	1.24
Longitudinal tensile strength (MPa)	2150	80
Longitudinal compressive strength (MPa)	1450	120
Shear strength (MPa)	1200	70

Periodic boundary conditions were applied.

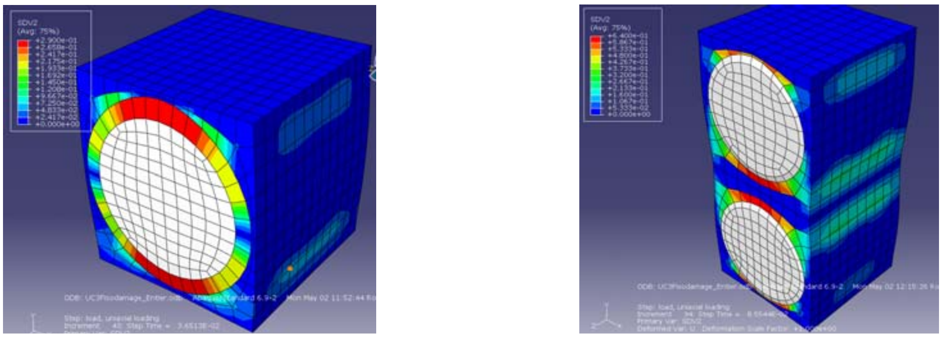

Figure 3. – Degradation of shear modulus at microstructure.

This model will be as initial approximation for multi-scale numerical simulation.

4.3 Orthotropic plies

In case of orthotropic plies the damage model of [13] was implemented in UMAT subroutine. Constitutive equation written as:

$$C = S^{-1} = \begin{bmatrix} \frac{1}{E_1(1-d_f)} & \frac{-\upsilon_{12}}{E_1(1-d_f)} & \frac{-\upsilon_{12}}{E_1(1-d_f)} & & & \\ \frac{-\upsilon_{12}}{E_1(1-d_f)} & \frac{1}{E_2(1-d_m)(1-d_t)} & \frac{-\upsilon_{23}}{E_2(1-d_t)} & & & \\ \frac{-\upsilon_{12}}{E_1(1-d_f)} & \frac{-\upsilon_{23}}{E_2(1-d_t)} & \frac{1}{E_2(1-d_m)} & & & \\ & & & \frac{1}{G_{23}(1-d_s)(1-d_m)} & & \\ & & & & \frac{1}{G_{12}(1-d_m)} & \\ & & & & & \frac{1}{G_{12}(1-d_s)(1-d_m)} \end{bmatrix} \quad (12)$$

Where d_f - fiber damage; d_m - damage matrix; d_t- transverse.

Table 2. Properties composites ply.

Name	E_1(GPa)	E_2(GPa)	υ_{12}	υ_{23}	G_{12}(GPa)
Value	148	9.57	0.356	0.49	4.5

On Figure 4 fiber damage and matrix damage are presented. Two types of geometrical models have been investigated: on Figure 4 – open hole specimen under tension loading and on Figure 5 – biaxial specimen under tension loading.

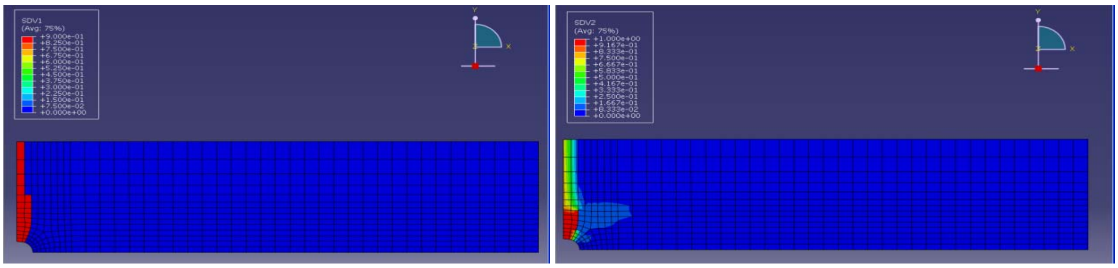

Figure 4. – Fiber damage and matrix damage at plies level. [0/90]s composites.

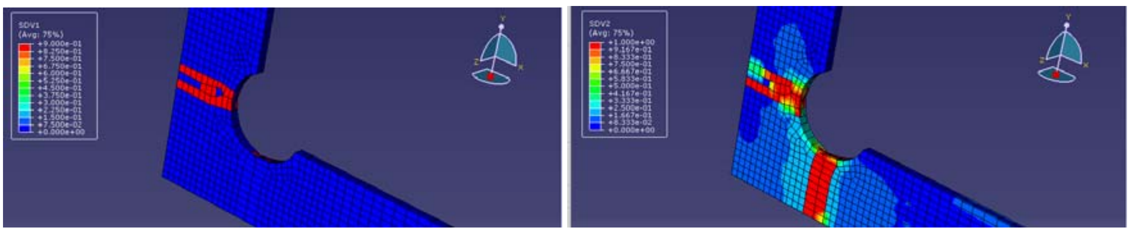

Figure 5. – Fiber damage and matrix damage at plies level in case of biaxial loading condition. [0/90]s composites.

4.4 SA life prediction

At the beginning SA elastic analysis of the structure has to be performed in ABAQUS to evaluate stress tensors components at every integration points and time increments. The geometry which is presented on the Figure 4 was modelled with using post-treatment software. Available results are micro deformation, number of cycle and damage obtained at each integration points.
General shape of these results is shown in Table 3.
SA should be applied for life composite under high cycle fatigue and allows us to obtain number of cycles when mesoscopic (in composites it is level of ply) cracks appear.

Table 3. Results of SA post-treatment.

Number of IP	Micro deformation	Damage	Life
1	0.005	0.0005	3500
⋮	⋮	⋮	⋮
102	0.02	0.001	2500
⋮	⋮	⋮	⋮
5600	0	0	inf

5 CONCLUSIONS

SA was developed based on continuous damage mechanics and micromechanics to predict the life of the composite structures which are subjected to high cycles fatigue loading.
This approach allows us to deal with different types of materials (as well as metals and composites). Managing parameters we can switch the calculation to: plastic, damage strong coupling and damage weak coupling. For instance when composite materials are considering the plastic deformation could be neglected but residual strain induced by damage inside specimen evaluated based on the damage potential surface.
Different types of simulation were performed: microscopic and macroscopic. Both of these models involved to multi-scale simulation. In order to reduce computational costs of multi-scale simulation the SA could be used.
SA was applied to simple tensile-tensile cycle fatigue analysis. Life, residual strains and damage are results of SA post-treatment. Number of cycles is characteristic of mesoscopic cracks initiation in composite materials.

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